Systems Engineering for Autonomous Systems
Defence Technology Centre

SEAS DTC

IF065

Topological Techniques to Assure Autonomous System Safety

The set of states of an autonomous system can be considered as a **space**, and this space is susceptible to topological analysis, making it possible to identify absolute bounds to the system's behaviour in the same way as energy considerations physically bound the trajectory of a projectile. **Topology** deals with the **connectedness** of points in a space, and can detect local features, such as holes, or global features, such as whether the space is in one piece or divided into many separate pieces. If the states of an autonomous system are equated to points in a state space, *holes* tell us that certain states cannot occur, while *disconnection* into separate pieces tells us that certain states cannot be reached from certain other states. For this research, the main tasks are (1) to show how to construct the state space and (2) to assess the information provided by various topological metrics.

Aim

To provide a mathematically rigorous way of verifying autonomous systems, i.e. ruling out unsafe behaviours as disallowed by the topology of the system's state space.

Approach

We view the system as a collection of relations between sets (e.g. sensors, actions). This is treated as a geometrical object and then analysed with the tools of algebraic topology.

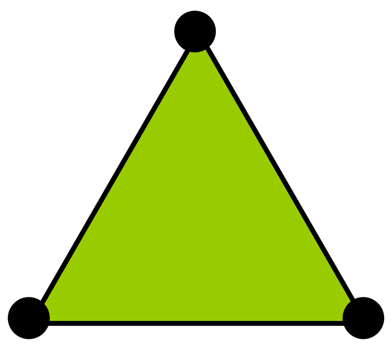
Outcomes

We used ideas of q-analysis, homology and co-homology to create original methods of identifying critical input subsets, assessing fault tolerance and designing procedural interlocks.

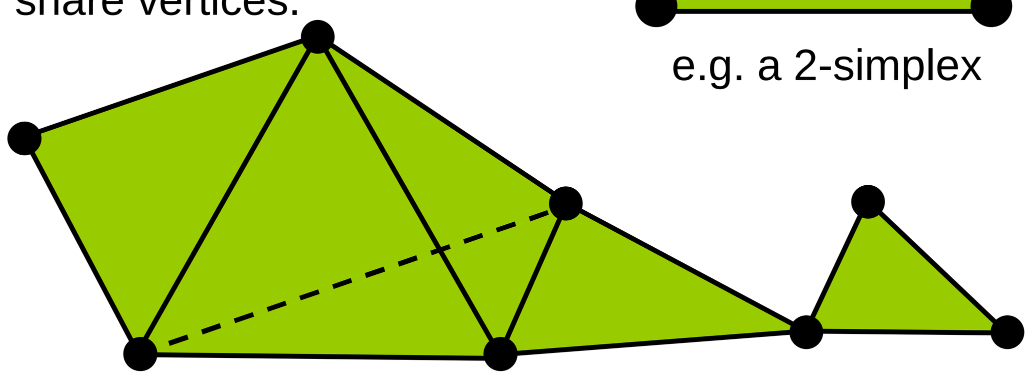
Simplicial complexes

An **n-simplex** is defined by n+1 linearly independent vertices and exists in n dimensions.

A **simplicial complex** is a collection of simplices that share vertices.

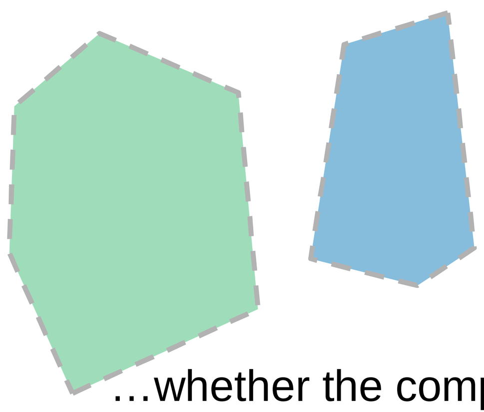


e.g. a 2-simplex



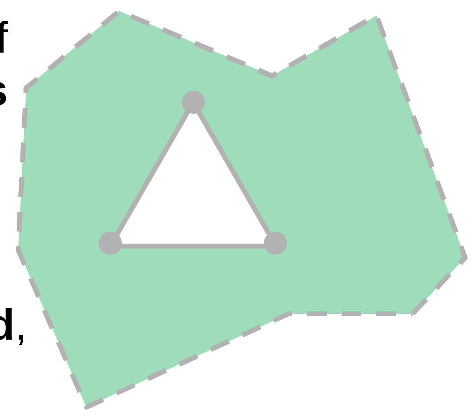
Algebraic topology

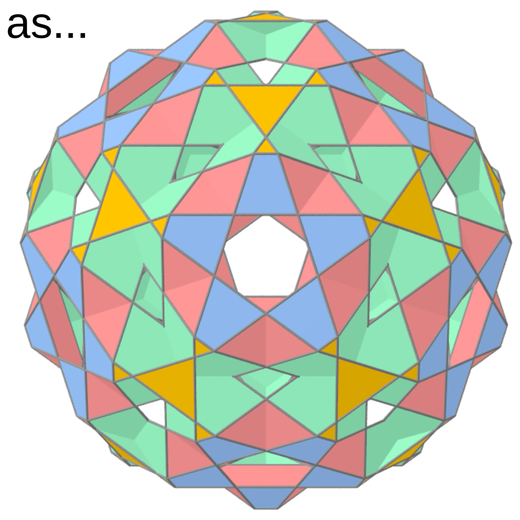
Algebraic topology analyses a simplicial complex to identify properties such as...



…whether the complex is **disconnected**, and how many pieces there are

…existence of **holes**





…number of **dimensions**

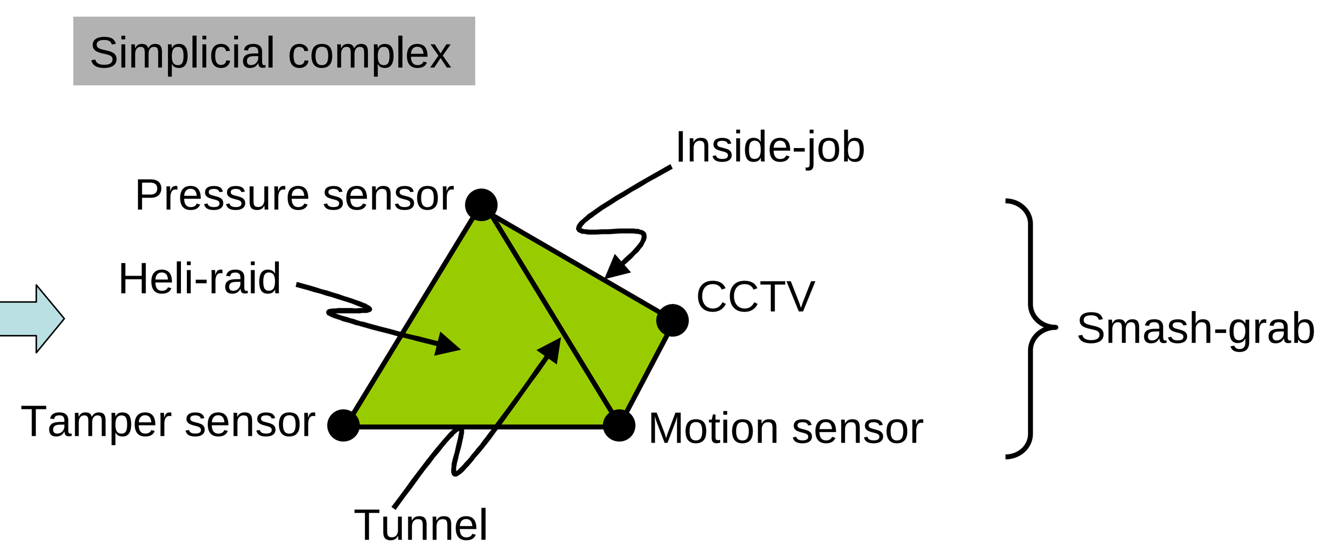
Typical simplicial complexes are high-dimensional and cannot be visualised. They must be handled algebraically, which also ensures mathematical rigour.

Structure of a relation

Work by C H Dowker (1950s), expanded by R H Atkin and others (1970s-1990s), showed how a **relation** between two **sets** can be used to define a simplicial complex, allowing the relation to be studied topologically.

Set of threats (columns); *Set of sensors* (rows)

Incidence matrix	Smash-grab	Heli-raid	Tunnel	Inside job
CCTV	1	0	0	1
Tamper	1	1	0	0
Motion	1	1	1	0
Pressure	1	1	1	1



This is important because the concepts of sets and relations are mathematically fundamental, and any problem can be expressed in such terms. E.g. the trajectory of a particle is a relation between a set of points and a set of times.

The example shown here is of a burglar-alarm system, where the relation is between a set of sensors and a set of threats they can detect.

Example: redundancy and resilience

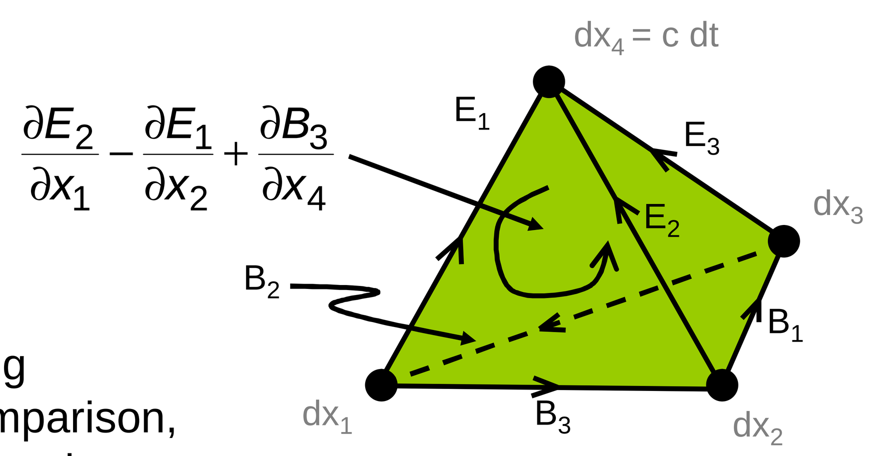
Analysing the topology of a relation e.g. between a set of sensors and a set of arcs gives information about the system's redundancy.

This can be used e.g. to design a token-passing algorithm so that coverage is maintained during sensor downtime.

Physics and topology

Physical laws such as Maxwell's equations can be derived from the properties of a suitable simplicial complex.

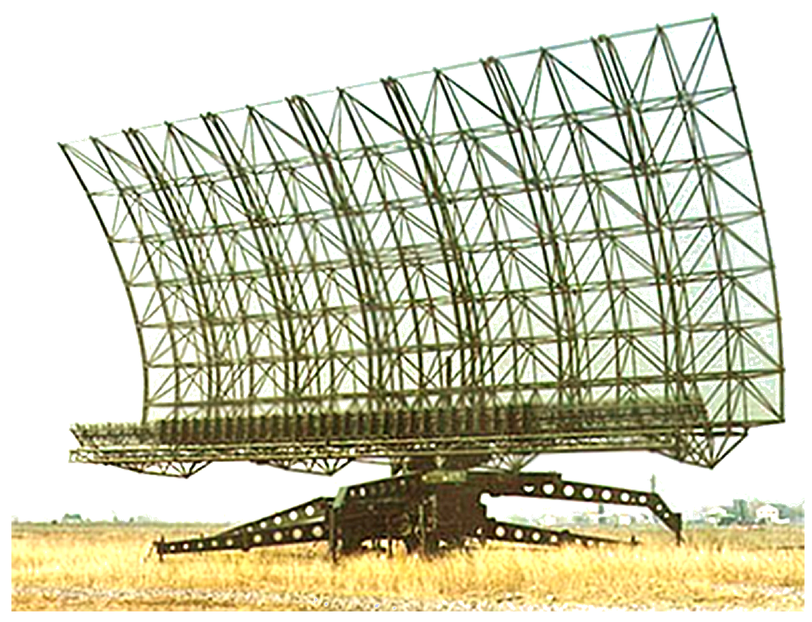
The simplicial complexes that describe engineering systems are highly irregular and convoluted in comparison, but the approach can be just as quantitative and precise.



Way forward

This project provided a glimpse of the possibilities. More research is needed to bring the methods to maturity in a real-world application.

For this we have devised a **sample scenario** based on an autonomous air defence system.



SELEX Systems Integration

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DTCs : Industry and the science base working in partnership with UK MoD to develop innovative defence technology

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